MATHEMATICAL DESCRIPTION OF THE TYPE FLIGHT MODEL OF AN AS532AL COUGAR HELICOPTER WITH AN EXTERNAL FIRE EXTINGUISHING DEVICE OF THE "BAMBI BUCKET BB4453" TYPE FOR EXTINGUISHING FIRES

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Abstract: *This article presents an analytical description of the model of a helicopter with an external fire extinguishing device in flight for extinguishing fires. The research is related to the helicopters used for this purpose by the Air Force of the Republic of Bulgaria AS532AL Cougar with an external fire extinguishing device of the Bambi Bucket "BB4453" type.*

МАТЕМАТИЧЕСКИ МОДЕЛ НА ТИПОВ ПОЛЕТ НА ХЕЛИКОПТЕР AS532AL COUGAR ЗА ГАСЕНЕ НА ПОЖАРИ С ВЪНШНО ПОЖАРОГАСИТЕЛНО УСТРОЙСТВО ОТ ТИПА "BAMBI BUCKET BB4453"

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Ключови думи: *хеликоптер, външно пожарогасително устройство, аеродинамични характеристики*

Резюме: *В настоящия материал е представен аналитично описание на модела на хеликоптер с външно пожарогасително устройство в полет за гасене на пожари. Изследванията са свързани с използваните за тази цел хеликоптери от Военновъздушните сили на Република България AS532AL Cougar с външно пожарогасително устройство от типа Bambi Bucket "BB4453".*

Introduction

The mathematical description of each object must ensure the implementation of the basic principles of mathematical models [1] and have all the necessary elements of the model [2]. In order to fulfill the task of mathematical description of the flight of an aircraft, the main principle is considered to be the consistency and adequacy of the obtained results, which requires that the conditions of applicability of the model be formulated with the utmost precision and that laws and regularities be used by scientific theory and practice over time. In this way, the necessary elements of the model receive completely clear and specific boundaries: a description of the basic assumptions and a closed mathematical description, including the equations of motion, the equations of the relationships between the elements, the numerical and functional characteristics of the aircraft and its control system.

Mathematical description of the type flight model of an AS532AL Cougar helicopter with an external fire extinguishing device for extinguishing fires

The central element of the model should be a system of equations of motion derived from the basic theorems of rigid body dynamics and kinematic relations [3]. In this paper, a system is considered that includes an AS532AL Cougar helicopter and an external fire extinguishing device of the “Bambi Bucket BB4453” type [4]. At the same time, the “Helicopter–External Fire Extinguishing Device” system is a very complex system and goes beyond the concept of a rigid body, which requires the compilation of several additional relations that depend to a significant extent on the assumptions made. For this reason, it is necessary to formulate the basic assumptions for the considered objects and their states at the very beginning, taking as a basis the need for solving the most important practical problems:

- the helicopter with one main propeller and one tail (stabilizing) propeller AS532AL Cougar is assumed in the description as an absolutely rigid body;
- the external fire extinguishing device “Bambi Bucket BB4453” with the so-called “spider web”, consisting of 24 short ropes (considered to be massless, absolutely rigid and inelastic in all directions) is assumed to be an absolutely rigid body;
- the attachment of “BB4453” to the AS532AL Cougar helicopter is carried out with a central cable, which is also assumed to be absolutely rigid and inelastic in all directions. The central cable is attached at one point to the “spider web” (24 short ropes), and the attachment at these two points is assumed to be performed using an ideal spherical joint. This allows, when describing the interaction of “Bambi Bucket BB4453” with the helicopter, to take into account only the tension force of the central cable;
- the aerodynamic characteristics of the helicopter, the characteristics of the forces and moments of its propellers, the characteristics of its control system, are considered to be set by the designer/manufacturer;
- the aerodynamic characteristics of the external fire extinguishing device are considered to be known;
- the mass and moments of inertia of the helicopter and the external fire extinguishing device remain known and constant in the considered time intervals.

Diagram of forces and moments acting in the “Helicopter–External Fire Extinguishing Device” system

Fig. 1 presents a diagram of the forces and moments acting in the “Helicopter–External Fire Extinguishing Device” system. It adopts the following notations:

r_{TP} – radius vector from the origin of the associated coordinate system to the point of application of the forces and moments from the tail propeller thrust;

r_{cc} – radius vector from the origin of the connected coordinate system at the point of attachment of the central cable to the helicopter;

r_{sw} – radius vector from the point of attachment of the central cable of the helicopter to the point of attachment of the cable to the end of the "spider web";

r_{ed} – radius vector from the center of mass of the “Bambi Bucket BB4453” at the point of attachment of the device to the end of the “spider web”;

P_{MP} – thrust of the main propeller;

P_{TP} – tail propeller thrust;

M_{MP} – moment of thrust of the main propeller;

M_{TP} – moment of the tail propeller thrust;

$M.g$ – helicopter weight;

M – mass of the helicopter;

m_{ed} – mass of the external fire extinguishing device;

$m_{ed}.g$ – weight of the external fire extinguishing device;

R_A – aerodynamic force acting on the helicopter;

$R_{A.ed}$ – aerodynamic force acting on the external fire extinguishing device;

M – aerodynamic moment acting on the helicopter;

M_{ed} – aerodynamic moment acting on the external fire extinguishing device;

T – tension force of the central cable of the external fire extinguishing device.

V – helicopter ground speed vector;

V_{ed} – ground speed vector of the external fire extinguishing device;

ω – angular velocity vector of the helicopter;

ω_{ed} – angular velocity vector of the external fire extinguishing device;

$$I_A = \begin{vmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{vmatrix}$$ – helicopter inertia tensor;

$$I_{A.ed} = \begin{vmatrix} I_{x.ed} & -I_{xy.ed} & -I_{xz.ed} \\ -I_{xy.ed} & I_{y.ed} & -I_{yz.ed} \\ -I_{xz.ed} & -I_{yz.ed} & I_{z.ed} \end{vmatrix}$$ – tensor of inertia of the external fire extinguishing device.

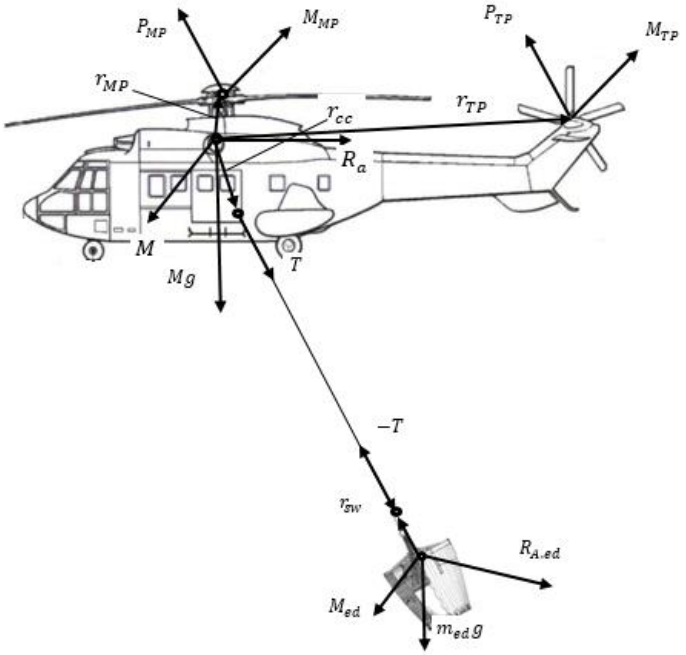


Fig. 1. Diagram of forces and moments acting in the system "Helicopter–External fire extinguishing device"

Within the framework of the assumptions made above, the complete system of differential equations for the motion of the helicopter with the external fire extinguishing device in vector form can include the following equations:

- force equation for the helicopter:

(1) $M\frac{dV}{dt} = P_{MP} + P_{TP} + R_A + Mg + T$;

- equation of moments for the helicopter:

(2) $I_A\frac{d\omega}{dt} = M_{MP} + M_{TP} + M + r_{MP}\,\mathrm{x}\,P_{MP} + r_{TP}\,\mathrm{x}\,P_{TP} + r_{sw}\,\mathrm{x}\,T$;

- equation of the kinematic relationships of the linear velocities of the center of mass of the helicopter:

(3) $\frac{d\rho}{dt} = V_{sw}$,

where ρ is the position vector of the helicopter's center of mass relative to the ground;

- equation of forces of the external fire extinguishing device:

(4) $m\frac{dV_{ed}}{dt} = R_{A.ed} + mg - T$;

- equation of moments for the external fire extinguishing device:

(5) $I_{A.ed}\frac{d\omega_{ed}}{dt} = M_{ed} - r_{ed}\,\mathrm{x}\,T$;

- equation of the kinematic relationships of the linear velocities of the center of mass of the external fire extinguishing device:

(6) $\frac{d\rho_{ed}}{dt} = V_{sw}$,

where ρ_{ed} is the position vector of the center of mass of the external fire extinguishing device relative to the ground surface;

- the kinematic conditions for the vectors r_{MP}, r_{TP}, r_{sw}, r_{ed}, which are constant in length (their variation is completely determined by the vectors ω and ω_{ed}:

(7) $\frac{dr_{MP}}{dt} = \omega \text{ x } r_{MP}$,

(8) $\frac{dr_{TP}}{dt} = \omega \text{ x } r_{TP}$,

(9) $\frac{dr_{sw}}{dt} = \omega \text{ x } r_{sw}$,

(10) $\frac{dr_{ed}}{dt} = \omega \text{ x } r_{ed}$.

Thus, the system of 10 vector differential equations determines 10 vectors: V, V_{ed}, ω, ω_{ed}, r_{MP}, r_{TP}, r_{sw}, r_{ed}, ρ, ρ_{ed}, if additional relations are given that determine [5, 6]:

- the forces and moments of the helicopter propeller thrust;
- the aerodynamic forces and moments of the helicopter and the external fire extinguishing device;
- the masses and moments of inertia of the helicopter and the external fire extinguishing device;
- the tension force of the central cable for the external fire extinguishing device.

To determine the forces and moments of the helicopter propeller thrust using systems of equations during the flight process, it is necessary to know not only the linear but also the angular coordinates, i.e. the orientation of the helicopter and the external fire extinguishing device. Therefore, the following equations must be added to the mathematical description for the calculations:

- the equation for the kinematic relationships between the angular velocities of the helicopter:

(11) $\frac{d\Phi}{dt} = \omega$,

where $\Phi = \{\vartheta, \gamma, \psi\}$ – the helicopter's angular orientation vector;

- the equation for the kinematic relationships between the angular velocities of the external fire extinguishing device:

(12) $\frac{d\Phi_{ed}}{dt} = \omega_{ed}$,

where $\Phi_{ed} = \{\vartheta_{ed}, \gamma_{ed}, \psi_{ed}\}$ – vector of the angular orientation of the external fire extinguishing device.

Depending on these coordinates, as well as on the angular velocities, at any moment of the flight time the necessary forces and moments of the helicopter propellers can be set. This is achieved with the help of the piloting model (of the control actions of the human from the flight crew, or from the autopilot from the control system), which set deviations in the controls depending on the current necessary and available needs, set as a function of the type $f(\rho, \rho_{ed}, V, V_{ed}, \Phi, \Phi_{ed}, \omega, \omega_{ed})$.

To determine the aerodynamic forces and moments of the helicopter and the external fire extinguishing device, it is necessary to know their aerodynamic characteristics. The assumption was made above that they are set by the designer/manufacturer of the equipment. However, since the coefficients of the aerodynamic forces and moments depend on the angles of attack and glide, their determination must also be added when determining the mathematical description. This is done immediately after determining these angles [7]. For this purpose, the components of the velocity vector in the associated coordinate system, in which the force equations are usually integrated, are used:

(13) $\alpha = arctg \frac{-V_{sw.y} + W_y}{V_{sw.x} - W_x}$,

(14) $\beta = arsin \frac{V_{sw.x} - W_x}{|V_{sw} - W|}$.

Formula (13) and Formula (14) allow the wind speed vector W to be taken into account, as it is added to each component of the speed vector.

According to the assumptions made, the masses and moments of inertia of the helicopter and the external fire extinguishing device are assumed to be known and constant.

The tension force in the central cable cannot be determined by direct differential equations. In this case, it is necessary to use algebraic relations for the type of connections existing in the mechanical system under study. Such are the relations between the central cable and the tension force in it:

- geometric condition for the central cable of the external fire extinguishing device:

(15) $r_{cc} = \rho_{ed} - \rho - r_{sw} + r_{ed}$;

- condition for constancy of the length of the central cable (in scalar form):

(16) $|r_{cc}| = |r_{cc.0}|$;

- condition for collinearity of the central cable and the tension force in it (two scalars):

(17) $\frac{r_{cc.x}}{T_x} = \frac{r_{cc.y}}{T_y} = \frac{r_{cc.z}}{T_z}$.

Conclusion

In the algorithm for mathematical description discussed above, it is necessary to select the value of the modulus of the central cable tension force vector $|T|$ in several successive iteration cycles in such a way that condition (16) is fulfilled. Since the relation (15) – (17) contains six equations for six components of the two vectors r_{cc} and T, since the general system of equations is closed [7], the mathematical description of the helicopter flight model with the external fire extinguishing device is complete.

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